

Thermal Equilibration and Expansion in Nucleus-Nucleus Collisions at the AGS

P. Braun-Munzinger, J. Stachel, J. P. Wessels and N. Xu*

Department of Physics

State University of New York at Stony Brook

Stony Brook, New York 11794 – 3800

(February 9, 2008)

Abstract

The rather complete data set of hadron yields from central Si + A collisions at the Brookhaven AGS is used to test whether the system at freeze-out is in thermal and hadro-chemical equilibrium. Rapidity and transverse momentum distributions are discussed with regards to the information they provide on hydrodynamic flow.

The goal of the ultra-relativistic heavy-ion program at the BNL AGS and CERN SPS is to study highly excited and dense nuclear matter and possibly the transition from hot and dense hadronic matter to deconfined quark-matter with restored chiral symmetry. While future collider experiments will probe a hot quark-gluon plasma with low net baryon density, present fixed target experiments create matter, possibly quark-matter, at very high baryon density and moderate temperature.

The present paper is following up on an earlier suggestion by some of us [1], based on the first AGS and SPS data, that a high degree of thermalization is reached and that there is evidence for hydrodynamic expansion of the created fireball. We now use the much larger set of data from central Si + A collisions at the AGS first to discuss quantitatively the

validity of a thermodynamic approach to interpret the data. Within this approach present data allow to determine the temperature of the system at the point when particles cease to interact strongly (freeze-out) as well as, via the net baryon density at freeze-out, the baryon chemical potential. The relatively large freeze-out temperature thus determined implies that even higher temperatures are reached earlier in the collision. In order to reach the freeze-out stage, the system has to expand considerably. Longitudinal and transverse spectra and, in particular, their mass dependence can yield information on the expansion velocity. This discussion forms the second part of this paper.

The present study starts on the following background:

- i) Production of transverse energy and the proton rapidity distribution after a central Silicon nucleus collision indicate a high degree of stopping [2–5,1,6].
- ii) Hadronic cascade codes that reproduce the hadron distributions at freeze-out require baryon densities in excess of five times normal nuclear matter density for an extended time, typically about 5 fm/c [6,7].
- iii) Data on pion interferometry are consistent with the fireball created in Si + Pb central collisions having a large final transverse radius of 6.7 fm (2.7 times the Si transverse radius), and a duration of the pion emission of 9 fm/c [8].

For this fireball scenario we explore quantitatively the predictions of a consistent thermodynamic and hydrodynamic approach to describe the AGS data. Calculations using ideal gas thermodynamics have been reported before [9–11] and compared to particle yield ratios. We use basically the same formalism as [9,10]. However, the authors of [9] had a much smaller set of early AGS and SPS data to compare to and our philosophy in fixing the model parameters is different. In contrast to [11], we use chemical equilibrium throughout combined with strangeness conservation.

To describe particle distributions and abundance at freeze-out, we need to treat the thermodynamics of the system only after the initially hot and dense fireball has expanded and reached the freeze-out density. In every co-moving restframe the system is therefore described by a grand canonical ensemble of non-interacting fermions and bosons in equilib-

rium at freeze-out temperature T . For an infinite volume the particle number densities are given as integrals over particle momentum p :

$$\rho_i^0 = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_b B_i - \mu_s S_i)/T] \pm 1} \quad (1)$$

where g_i is the spin-isospin degeneracy of particle i , E_i , B_i and S_i are its total energy in the local restframe, baryon number and strangeness, and μ_b and μ_s are the baryon and strangeness chemical potentials ($\hbar=c=1$ unless otherwise noted). For a system of finite size the argument of the integral in equation (1) has to be multiplied [12] by a correction factor which we evaluate for a spherical volume with radius R . We also apply the excluded volume correction [10] to take into account the volume occupied by individual baryons and mesons with radii of 0.8 and 0.6 fm.

The temperature range relevant for the present study is 0.10 - 0.15 GeV which sets the scale for the mass range of particles to be considered. We include strange and non-strange mesons up to a mass of 1.5 GeV and baryons up to 2 GeV. Results change by less than 10% if the mass range is restricted to mesons lighter than 1 GeV and baryons lighter than 1.5 GeV, indicating the sensitivity to the mass cut. We have omitted hyperons with strangeness 2 or larger *and* mass above 1.6 GeV since their yields and decays will not impact on any of the presently observed quantities. It is very important, however, to treat particles and antiparticles as well as different isospin states evenly, i.e. to include all states at a given energy, since observables like the $\bar{\Lambda}/\Lambda$ or \bar{p}/p ratio are strongly affected by feeding from higher states.

The range of temperatures to be considered is driven by the experimental observation [13] of the occupation probability of the $\Delta(1232)$ resonance in E814: For a system in thermal equilibrium, temperatures of 0.12 - 0.14 GeV are consistent with the observed abundance. The baryon chemical potential is constrained by the measured pion to nucleon abundance as well as the density of the system at freeze-out [8]. We choose a value of $\mu_b = 0.54$ GeV. For a given temperature and baryon chemical potential the strangeness chemical potential is fixed by strangeness conservation. In particular, for $T = 0.120$ and 0.140 GeV one obtains

$\mu_s = 0.108$ and 0.135 GeV. Using these input parameters and equation (1) we find primary particle densities.

For comparison to experimental data one needs to consider decay and feeding. We use all the known branching ratios as given in [14] as well as symmetry and phase space arguments for unknown branching ratios. To illustrate the importance of treating the feeding properly we note that, at $T = 0.14$ GeV, the primary pion yield is tripled and the nucleon yield is doubled by feeding from higher resonances.

Flow effects discussed below do not affect angle integrated particle densities, but may severely change particle densities and density ratios at fixed rapidity. We, therefore, compare predictions of the thermal model to experimental quantities integrated over transverse momentum p_t and rapidity y . The y integration is justified since even a rather well localized thermal source in the center of the colliding nuclei is spread out in y because of the width of the space-rapidity correlation and the natural width in y of a thermal source. Also the data cover at most 2 units of y and all include midrapidity.

In Table I all currently available experimental data on particle ratios measured in central Si + Au(Pb) collisions are compared to the corresponding ratios calculated for two temperatures, 0.12 and 0.14 GeV. Overall good agreement is found. Although the yields vary over three orders of magnitude, all particle ratios are reproduced to better than a factor of two. While our choice for the temperature range considered is driven by the $\Delta(1232)$ abundance and not the yield ratios in Table I, the range considered nevertheless gives the best overall agreement; the experimental $d/(p+n)$, K/π , $\Lambda/(p+n)$ and ϕ/K ratios favor a slightly lower temperature, the \bar{p}/p and $\bar{\Lambda}/\Lambda$ ratios are bracketed by the range considered, the K^+/K^- and Ξ^-/Λ ratios favor higher T .

The choice of μ_b is a trade-off between the observed pion to nucleon ratio and the particle density at freeze-out. At a given μ_b , absolute particle densities predicted by the model are very sensitive to the exact freeze-out temperature; for temperatures of $T = 0.12$ (0.14) GeV and $\mu_b = 0.54$ GeV, densities for nucleons and pions of $\rho_n = 0.070$ (0.13)/fm³ and $\rho_\pi = 0.09$ (0.17)/fm³ are obtained. These numbers should be compared with $\rho_n^{exp} = 0.058/\text{fm}^3$

and $\rho_{\pi}^{exp} = 0.063/\text{fm}^3$ recently estimated [8] for Si + Pb central collisions. Especially for the lower temperature the agreement for the absolute densities is surprisingly good. From the data displayed in Table I there is no indication that strangeness is not in chemical equilibrium.

In the following we will discuss whether the experimental rapidity and transverse momentum spectra are consistent with the prediction of such a thermodynamic model allowing for possible flow effects. Equation (1) implies random, isotropic emission. Since the temperature at freeze-out exceeds 100 MeV, we use the Boltzmann approximation. Transformed into rapidity y and transverse momentum p_t this implies:

$$E \frac{d^3N}{d^3p} \propto E \exp(-E/T) = m_t \cosh(y) \exp(-m_t \cosh(y)/T) \quad (2)$$

for a particle with mass m and transverse mass m_t . All kinematical variables are evaluated in the nucleon-nucleon center of momentum frame. Integrating over m_t , one obtains:

$$\frac{dN_{iso}}{dy} \propto m^2 T (1 + 2\chi + 2\chi^2) \exp(-1/\chi) \quad (3)$$

with $\chi = T/(m \cosh(y))$. For pions with $m \approx T$, this distribution is close to that for massless particles, *i.e.* proportional to $1/\cosh^2(y)$. For heavier particles isotropic emission implies a strong narrowing of the distribution, in contradistinction to experimental observations. This is demonstrated in Fig. 1, where experimental rapidity distributions [3,4,16,21] for central collisions of Si + Al are compared to predictions of the isotropic thermal model (solid line). This rather small system was chosen since it is the only symmetric system where data for particle distributions are published. The calculations were performed for $T = 0.12$ GeV; the situation is very similar at 0.14 GeV. Obviously the agreement between the isotropic model calculation and data is poor. Much better agreement between data and calculations is obtained by introducing a common collective flow velocity in longitudinal direction. Following [19,20] we superpose individual isotropic thermal sources within a rapidity interval $[-y'_{max}, y'_{max}]$:

$$\frac{dN}{dy} = \int_{-y'_{max}}^{y'_{max}} dy' \frac{dN_{iso}(y - y')}{dy'}. \quad (4)$$

The integration limit y'_{max} confines boost invariance to a finite rapidity interval; we treat it as a free parameter to determine the amount of flow required by the data. The results of calculations for such a longitudinally expanding fireball are shown as dashed lines in Fig. 1. Good overall agreement with the data is found for $y'_{max} = 1.15$. Averaged over the forward (backward) portion of the flat distribution (4) this corresponds to $\langle y' \rangle = 0.58$ (-0.58) and a mean longitudinal flow velocity of $\langle \beta_l \rangle = \tanh(\langle y' \rangle) = 0.52$. We note that, although the normalization of the calculated curves has been adjusted for each particle species separately, the normalization factors in principle could have been taken from our calculations above. This would imply, *e.g.* an upward shift of 20%, 37% and 60% for p, π^+ , and K^+ . Considering the uncertainty in the freeze-out volume, this agreement is remarkably good.

Rapidity distributions of d, Λ , p, K are not strongly affected by resonance decays [20]. The situation may be different, however, for pions: as demonstrated elsewhere [13], at AGS energies about 1/3 of all pions originate from decay of the $\Delta(1232)$ resonance. This will also widen somewhat the pion rapidity distributions, and may account for part of the effect flow has on the pion distributions.

The width of the rapidity distribution for protons may also be an indication for incomplete stopping in this rather small system [3]. From the difference in the p and Λ distributions one can infer the degree of transparency. The Λ distribution in Fig. 1 does not support $\langle \beta_l \rangle$ larger than 0.52 indicating that the protons are slowed down 0.2 units of rapidity less than required for full stopping. Overall, thermalization plus longitudinal flow provide a fairly consistent description of all rapidity distributions already for the relatively small system Si + Al.

Assuming that the longitudinal and transverse motion of the thermal source are decoupled, flow effects can be incorporated into transverse momentum spectra following [20]:

$$\frac{dN}{m_t dm_t} \propto \int_0^R r dr m_t I_0\left(\frac{p_t \sinh(\varrho)}{T}\right) K_1\left(\frac{m_t \cosh(\varrho)}{T}\right) \quad (5)$$

with Bessel functions I_0 and K_1 , a parameter $\varrho = \tanh^{-1}(\beta_t)$ and a transverse velocity profile of $\beta_t(r) = \beta_t^{max}(r/R)^\alpha$. The flow parameter β_t^{max} determined from data for $(m_t - m) >$

$0.3 \text{ GeV}/c^2$ is not very sensitive to the magnitude of the parameter α ; we have used $\alpha = 1$ in the subsequent calculations. Since we will focus our analysis on Si + Au data, we choose for the transverse freeze-out radius of the system the value $R = 6.7 \text{ fm}$ [8].

Rather than exploring the parameter space (T, β_t^{max}) describing the data, we fix the freeze-out temperature between 0.12 and 0.14 GeV. We then determine whether the data can be consistently described with one common transverse expansion velocity by comparing results of calculations using eq. (5) with data [4] near midrapidity.

To determine best fit values for β_t^{max} , we restrict the fit to $(m_t - m) > 0.3 \text{ GeV}/c^2$. In this range resonance decays yield negligible changes in spectra for d, p, and K. For pions inclusion of resonance decays makes a significant change at low $(m_t - m)$ values. We have, therefore, included resonance decays into the calculation of transverse momentum spectra by numerical simulation of two-body and three-body decays of the dominant resonances.

The results are presented in Fig. 2. Remarkable agreement between data and calculations is obtained for $\beta_t^{max} = 0.58$ (0.50) at $T = 0.12$ (0.14) GeV, even for pion spectra at low m_t . This corresponds to average flow velocities of $\langle\beta_t\rangle = 0.39$ (0.33) and implies expansion times of the order of 12 fm/c. Independent support for the presence of flow effects was recently obtained [22] in an analysis of azimuthal distributions of transverse energy in semi-central Au+Au collisions at AGS energies.

We have demonstrated that the presently available AGS data can be consistently described in a thermal model with chemical equilibrium and flow. This includes particle densities, ratios of produced particles, and rapidity and transverse momentum distributions. The thermal parameters describing freeze-out are $T = 0.12 - 0.14 \text{ GeV}$, $\mu_b = 0.54 \text{ GeV}$, $\langle\beta_t\rangle = 0.52$, and $\langle\beta_t\rangle = 0.39 - 0.33$. Earlier times in the evolution of the fireball need to be probed with different observables to determine the equation of state during the high density phase.

This work was supported by the NSF. One of us (J.P.W.) is supported by the A. v. Humboldt Foundation as a F. Lynen fellow.

* present address: P-25, MS D456, LANL, Los Alamos, NM 87545.

REFERENCES

- [1] J. Stachel and P. Braun-Munzinger, Phys. Lett. **B216**, 1(1989).
- [2] J. Barrette *et al.*, E814 Coll., Phys. Rev. Lett. **64**, 1219(1990) and Phys. Rev. Lett. **70**, 2996(1993).
- [3] J. Barrette *et al.*, E814 Coll., Z. Phys., **C59**, 211(1993); J. Barrette *et al.*, E814 Coll., Phys. Rev. **C50**, in print, preprint SUNY-RHI-94-9.
- [4] T. Abbott *et al.*, E802 Coll., Phys. Rev. **C50**, 1024(1994).
- [5] S. E. Eiseman *et al.*, E810 Coll., Phys. Lett. **B292**, 10(1992).
- [6] H. Sorge, A. von Keitz, R. Mattiello, H. Stöcker, and W. Greiner, Phys. Lett. **B243**, 7(1990); H. Sorge, R. Mattiello, H. Stöcker, and W. Greiner, Phys. Lett. **B271**, 37(1991).
- [7] Y. Pang, T. Schlagel, and S. K. Kahana, Phys. Rev. Lett. **68**, 2743(1992); S. K. Kahana, in Proc. Int. Workshop “Heavy Ion Physics at the AGS ’93”, G. S. F. Stephans, S. G. Steadman, W. L. Kehoe, eds., MITLNS-2158, p. 263.
- [8] J. Barrette *et al.*, E814 Coll., Phys. Lett. **B333**, 33(1994); N. Xu, E814 Coll., Nucl. Phys. **A566**, 585c(1994).
- [9] J. Cleymans, H. Satz, E. Suhonen, and D. W. von Oertzen, Phys. Lett. **B242**, 111(1990); N. J. Davidson, H. G. Miller, R. M. Quick, and J. Cleymans, Phys. Lett. **B255**, 105(1991); N. J. Davidson, H. G. Miller, and D. W. von Oertzen, Phys. Lett. **B256**, 554(1991).
- [10] J. Cleymans and H. Satz, Z. Physik **C57**, 135(1993).
- [11] J. Letessier, J. Rafelski, A. Tounsi, Phys. Lett. **B328**, 499(1994).
- [12] H. R. Jaqama, A. Z. Mekjian, and L. Zamick, Phys. Rev. **C29**, 2067(1984).
- [13] T. K. Hemmick, E814 Coll., Nucl. Phys. **A566**, 435c(1994); J. Barrette *et al.*, E814

- Coll., sub. Phys. Lett. **B**, Nov. 1994, preprint SUNY-RHI-94-10;
- [14] Review of Particle Properties, Phys. Rev. **D50**, 1173-1926(1994).
- [15] G. S. F. Stephans, E802 Coll., Nucl. Phys. **A566**, 269c(1994).
- [16] S. E. Eiseman *et al.*, E810 Coll., Phys. Lett. **B297**, 44(1992).
- [17] S. E. Eiseman *et al.*, E810 Coll., Phys. Lett. **B325**, 322(1994).
- [18] A. Aoki *et al.*, E858 Coll., Phys. Rev. Lett. **69**, 2345(1992).
- [19] A. N. Makhlin and Y. M. Sinyukov, Z. Phys. **C39**, 69(1988).
- [20] E. Schnedermann, J. Sollfrank, and U. Heinz, Phys. Rev. **C48**, 2462(1993); E. Schnedermann and U. Heinz, Phys. Rev. **C50**, 1675(1994).
- [21] T. K. Hemmick, E814 Coll., in Proc. Int. Workshop “Heavy Ion Physics at the AGS ’93”, G. S. F. Stephans, S. G. Steadman, and W. L. Kehoe eds., MITLNS-2158, p. 204.
- [22] J. Barrette *et al.*, E877 Coll., Phys. Rev. Lett. **73** 2532(1994).

TABLES

TABLE I. Particle ratios calculated in a thermal model for two different temperatures, baryon chemical potential $\mu_b = 0.54$ GeV and strangeness chemical potential μ_s such that overall strangeness is conserved, in comparison to experimental data (with statistical errors in parentheses) for central collisions of 14.6 A GeV/c Si + Au(Pb).

Particles	Thermal Model		Data		
	$T=.120$ GeV	$T=.140$ GeV	exp. ratio	rapidity	ref.
$\pi/(p+n)$	1.29	1.34	1.05(5)	0.6 - 2.8	[4,3]
$d/(p+n)$	$4.3 \cdot 10^{-2}$	$5.8 \cdot 10^{-2}$	$3.0(3) \cdot 10^{-2}$	0.4 - 1.6	[4]
\bar{p}/p	$1.47 \cdot 10^{-4}$	$5.8 \cdot 10^{-4}$	$4.5(5) \cdot 10^{-4}$	0.8 - 2.2	[15]
K^+/π^+	0.23	0.27	0.19(2)	0.6 - 2.2	[4]
K^-/π^-	$5.0 \cdot 10^{-2}$	$6.2 \cdot 10^{-2}$	$3.5(5) \cdot 10^{-2}$	0.6 - 2.3	[4]
K_s^0/π^+	0.14	0.16	$9.7(15) \cdot 10^{-2}$	2.0 - 3.5	[16,4,21]
K^+/K^-	4.6	4.3	4.4(4)	0.7 - 2.3	[4]
$\Lambda/(p+n)$	$9.5 \cdot 10^{-2}$	0.11	$8.0(16) \cdot 10^{-2}$	1.4 - 2.9	[16,4,3]
$\bar{\Lambda}/\Lambda$	$8.8 \cdot 10^{-4}$	$3.7 \cdot 10^{-3}$	$2.0(8) \cdot 10^{-3}$	1.2 - 1.7	[15]
$\phi/(K^+ + K^-)$	$2.4 \cdot 10^{-2}$	$3.6 \cdot 10^{-2}$	$1.34(36) \cdot 10^{-2}$	1.2 - 2.0	[15]
Ξ^-/Λ	$6.4 \cdot 10^{-2}$	$7.2 \cdot 10^{-2}$	0.12(2)	1.4 - 2.9	[17]
\bar{d}/\bar{p}	$1.1 \cdot 10^{-5}$	$4.7 \cdot 10^{-5}$	$1.0(5) \cdot 10^{-5}$	2.0	[18]

FIGURES

FIG. 1. Rapidity distributions for central 14.6 A GeV/c Si+Al collisions [3,4,16,21] in comparison to isotropic thermal distributions at $T = 0.12$ GeV (solid lines) and distributions for a source at the same temperature expanding with $\langle\beta_t\rangle = 0.52$ (dashed lines).

FIG. 2. Experimental particle spectra [4] at $y = 1.3$ compared to calculated spectra for a source at $T = 0.12$ GeV expanding transversely with $\langle\beta_t\rangle = 0.39$ (left) and a source at $T = 0.14$ GeV and $\langle\beta_t\rangle = 0.33$ (right). The arrows indicate the beginning of the fit region. For details, including the treatment of resonance decays, see text.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9410026v2>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9410026v2>



